

SEMINAR WINTERSEMESTER 2019/20

HYPERKÄHLER GEOMETRY

Thursdays, 12-2pm, Room 016 in the Mensa Building

Hyperkähler geometry is a type of Riemannian geometry, based on quaternions. The holonomy group of a hyperkähler manifold is a subgroup of $Sp(n)$, and hence all hyperkähler manifolds are Ricci-flat.

Many examples of noncompact hyperkähler manifolds arise as moduli spaces of solutions to gauge-theoretic equations which arise from the dimensional reduction of the anti-self dual YangMills equations: instanton moduli spaces, monopole moduli spaces, spaces of solutions to Hitchin's self-duality equations on Riemann surfaces. Another class of examples are the Nakajima quiver varieties, which are of great importance in representation theory.

I propose to concentrate the seminar on a few chosen topics, e.g. gravitational instantons (i.e. complete 4-dimensional hyperkähler manifolds), Hilbert schemes of points, quiver varieties, hypertoric or hyperpolygon spaces.

Bibliography:

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6. J. Fisher and S. Rayan, 'Hyperpolygons and Hitchin systems", *Int. Math. Res. Notices* (2016), no. 6, 1839–1870.
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10. H. Nakajima, 'Instantons on ALE spaces, quiver varieties, and Kac-Moody algebras', *Duke Math. J.* 76 (1994), 365–416.
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