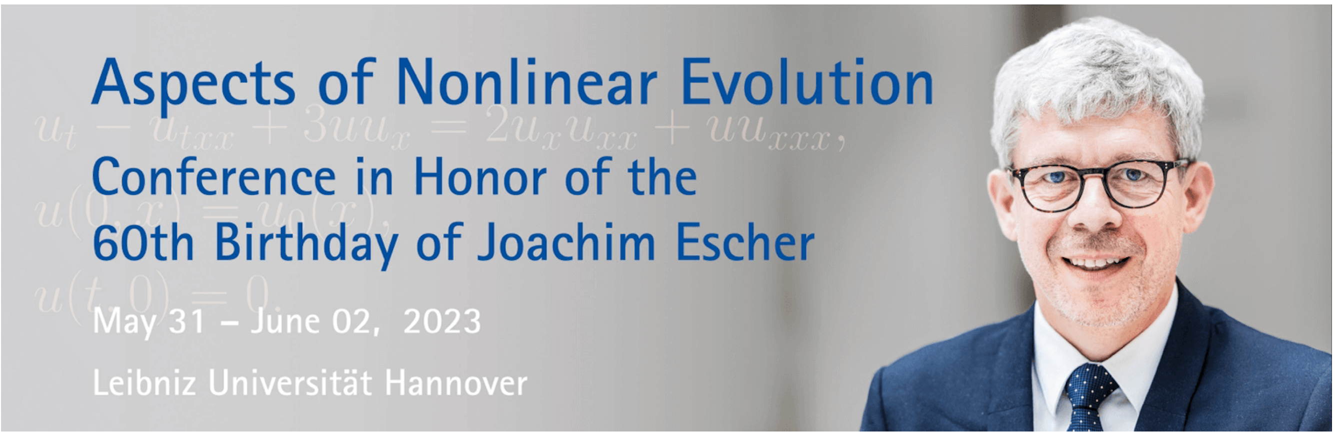

Book of Abstracts

From Forms to Non-linear Semigroups

Wolfgang Arendt

Universität Ulm
wolfgang.arendt@uni-ulm.de

Abstract

We present a variational method to obtain semigroups from forms. A prototype is the Dirichlet-to-Neumann operator. The method can be extended to non-linear semigroups where subgradients replace the form. Of great importance is a result on maximal regularity due to H. Brezis.

References:

J. Escher: The Dirichlet-to-Neumann Operator on continuous functions. Ann. Scuola Norm. Sup. Pisa 21 (1994) 235-266.

W. Arendt, A.F.M. ter Elst: The Dirichlet-to-Neumann Operator on $C(\Gamma)$. Ann. Scuola Norm. Pisa 20 (2020) 1169-1196.

W. Arendt, D. Hauer: Maximal L_2-regularity in non-linear gradient systems and perturbation by non-linear growth. Pure & Applied Analysis 2 (2020) 23-34.

Counterintuitive approximations

Christian Bär

Universität Potsdam
cbaer@uni-potsdam.de

Abstract

The Nash-Kuiper embedding theorem is a prototypical example of a counterintuitive approximation result: any short (but highly non-isometric) embedding of a Riemannian manifold into Euclidean space can be approximated by isometric C^1-embeddings. As a consequence, any surface can be isometrically C^1-embedded into an arbitrarily small ball in $\mathbb{R}^3$. For C^2-embeddings this is impossible due to curvature restrictions.

I will present a general result which allows for approximations by functions satisfying strongly overdetermined equations on open dense subsets. This will be illustrated by three examples: Lipschitz functions with surprising derivative, surfaces in 3-space with unexpected curvature properties, and a similar statement for abstract Riemannian metrics on manifolds. Our method is based on "cut-off homotopy", a concept introduced by Gromov in 1986.

This is based on joint work with Bernhard Hanke (Augsburg).

The evolution in time of extrema

(Colloquium Talk)

Adrian Constantin

Universität Wien
adrian.constantin@univie.ac.at

Abstract

We discuss the evolution in time of the maxima and minima of bounded regular solutions to well-posed partial differential equations. We show that the key result in this direction can be applied to study global existence in time as well as blow-up in finite time to nonlinear hyperbolic and parabolic partial differential equations.

On two new constructions of solitary waves of the nonlinear and nonlocally dispersive Whitham equation

Mats Ehrnström

NTNU Trondheim
mats.ehrnstrom@ntnu.no

Abstract

Solitary waves in dispersive and water wave equations are often constructed using either constrained minimisation or pertubative techniques around a trivial flow. In both cases, the resulting waves are typically small, because of nonlinear control. We present

here two new proofs for existence of solitary waves in the nonlinear and nonlocal evolution equation

$$u_t + Lu_x + uu_x = 0, \qquad \mathcal{F}(Lu)(\xi) = \left(\frac{\tanh(\xi)}{\xi}\right)^{1/2} \mathcal{F}u(\xi)$$

also called the Whitham equation. The first proof is based on a priori estimates of periodic waves of all heights, and uses a limiting argument in the periodic to obtain a family of solitary waves up to the highest wave. The second uses a maximisation technique perhaps not earlier used in the water wave setting, where the dispersive part of the energy functional is maximised whereas remaining terms are held as a constraint in an Orlicz space constructed directly for this purpose. That is in many respects an L^p-based maximisation technique. We find in the second work small and intermediate-sized waves, although not necessarily a highest solitary wave.

The first work is joint with K. Nik and C. Walker; the second with A. Stefanov and M. N. Arnesen.

On the interaction of mean curvature flow and diffusion on evolving hypersurfaces

Harald Garcke

Universität Regensburg

harald.garcke@mathematik.uni-regensburg.de

Abstract

We consider a geometric problem consisting of an evolution equation for a closed hypersurface coupled to a parabolic equation on this evolving surface. More precisely, the evolution of the hypersurface is determined by a scaled mean curvature flow that depends on a quantity defined on the surface via a diffusion equation. This system arises as a gradient flow of a simple energy functional. Assuming suitable parabolicity conditions, we derive short-time existence for the system. The proof is based on linearization and a contraction argument. For this, we parameterize the hypersurface via a height function and thus the system, originally defined on an evolving surface, can be transformed onto a fixed reference surface. The result is formulated in a classical sense, holds for the case of embedded and immersed hypersurfaces alike and provides an existence time independent of small changes in the initial surface. Afterwards, several properties of the solution are analyzed. Emphasis is placed on to what extent the surface in our setting evolves the same as for the usual mean curvature flow. To this end, we show that the surface area is strictly decreasing but give an example of a surface that exists for infinite times nevertheless. Moreover, mean convexity is conserved whereas convexity is not. Finally,

we construct an embedded hypersurface that develops a self-intersection in the course of time. Finally, we discuss how solutions can be computed numerically with the help of an evolving surface finite element discretization. We will discuss optimal error bounds and present numerical experiments illustrating the above discussed qualitative properties of the flow as well as the convergence behaviour.

The analytical part is a joint work with Helmut Abels and Felicitas Burger (both University of Regensburg) and the numerical part is joint work with Charlie Elliott (University of Warwick) and Balàzs Kovàcs (University of Regensburg).

The Curve Shortening Flow for Immersed Curves

Patrick Guidotti

University of California at Irvine
gpatrick@math.uci.edu

Abstract

In this talk we will revisit the Curve Shortening Flow for immersed curves and present some results about existence, special solutions, and a new numerical scheme for its solution.

Energy considerations for nonlinear water waves in the Equatorial f-plane

David Henry

University College Cork
d.henry@ucc.ie

Abstract

This talk presents recent results concerning excess energy densities for exact nonlinear water waves. The theory underlying ocean wave energy suffers from the fundamental limitation that most of the state-of-the-art is strongly contingent on invoking linear approximations. In this talk we consider the excess kinetic and potential energies for exact nonlinear equatorial water waves in the f-plane. An investigation of linear waves establishes that the excess kinetic energy density is always negative, whereas the excess potential energy density is always positive, for periodic travelling irrotational water waves in the steady reference frame. For negative wavespeeds, we prove that similar inequalities must also hold for nonlinear wave solutions. Characterisations of the various excess energy densities as integrals along the wave surface profile are also derived.

A Journey through the World of Incompressible Fluids and Geophysical Flows

(Colloquium Talk)

Matthias Hieber

TU Darmstadt
hieber@mathematik.tu-darmstadt.de

Abstract

Starting from the basic balance laws in physics for mass, momentum and energy, we discuss in this talk classical as well as recent aspects of the mathematical investigation of incompressible fluids ranging from Newtonian fluids and the equations of Navier-Stokes and Euler over geophysical flows including deterministic and stochastic forces and boundary conditions such as transport noise to coupled atmosphere-sea-ice-ocean models. Of concern are mathematical models representing these fluids, as well as fluid-structure interaction phenomena and free boundary value problems.

Equatorial internal waves

Rossen Ivanov

Technological University Dublin
rossen.ivanov@TUDublin.ie

Abstract

We analyse the nonlinear equations of motion for coupled equatorial surface and internal waves. These waves propagate along the Equator together with the so-called

Equatorial Undercurrent (EUC). We therefore take into account the wave-current interactions in the physical setting of azimuthal two-dimensional inviscid flows. The system is modelled with piecewise constant vorticity in a two-layer fluid with a flat bed and a free surface.

We derive a Hamiltonian formulation for the nonlinear governing equations that is adequate for structure-preserving perturbations, at the linear and at the nonlinear level. The fact that ocean energy is concentrated in the long-wave propagation modes motivates the pursuit of in-depth nonlinear analysis in the long-wave regime. The arising integrable Hamiltonian equations possess stable solution solitons, which could be detected by oceanographic measurements.

The Euler-Arnold equation on the diffeomorphism group

Boris Kolev

ENS Paris-Saclay
boris.kolev@math.cnrs.fr

Abstract

I will present an overview of joint works with Joachim Escher about geodesic flows of Sobolev–type metrics on the diffeomorphism group. These contributions include local and global existence results for these flows and the behaviour of the Riemannian exponential mapping.

Liouville-type results for the time-dependent three-dimensional water wave problem with an interface

Calin Martin

Universität Wien
calin.martin@univie.ac.at

Abstract

We consider time-dependent three-dimensional stratified water flows of finite depth with a free surface and an interface (separating two layers of constant and different densities). Under the assumption that the vorticity vectors in the two layers are constant,

non-vanishing, we prove that bounded solutions to the three-dimensional equations are essentially two-dimensional.

A new reformulation of the Muskat problem with surface tension

Anca Matioc

Universität Regensburg
anca.matioc@mathematik.uni-regensburg.de

Abstract

In this talk two formulas that connect the derivatives of the double layer potential and of a related singular integral operator evaluated at some density ϑ to the L_2-adjoints of these operators evaluated at the density ϑ' are used to recast the Muskat problem with surface tension and general viscosities as a system of equations with nonlinearities expressed in terms of the L_2-adjoints of these operators. An advantage of this formulation is that the nonlinearities appear now as a derivative. This aspect and abstract quasilinear parabolic theory are then exploited to establish a local well-posedness result in all subcritical Sobolev spaces $W^s_p(\mathbb{R})$ with $p \in (1, \infty)$ and $s \in (1 + 1/p, 2)$. (Joint work with Bogdan Matioc, Regensburg.)

The nonlocal mean curvature flow of periodic graphs

Bogdan Matioc

Universität Regensburg
bogdan.matioc@mathematik.uni-regensburg.de

Abstract

We discuss a model that describes the evolution of a periodic graph in a multidimensional setting by its fractional nonlocal mean curvature.This problem can be formulated in a suitable functional analytic framework as a quasilinear evolution problem which turns out to be of parabolic type. The well-posedness of the problem in all subcritical small

Hölder spaces follows then by using abstract theory available for such problems. Moreover, solutions which are initially close to an horizontal graph in the phase space exist globally in time and they converge exponentially fast towards a flat graph. Based on joint work with Christoph Walker.

Time-Periodic Waves for a Quasilinear Wave Equations Based on a Kerr-Nonlinear Maxwell Model

Wolfgang Reichel

Karlsruher Institut für Technologie
wolfgang.reichel@kit.edu

Abstract

We consider quasilinear wave equations $g(x)w_{tt} - \Delta w + h(x)(w^3)_{tt} = 0$ on $\mathbb{R} \times \mathbb{R}^d$ with $d = 1, 2$ which arise in the study of localized electromagnetic waves modeled by Kerr-nonlinear Maxwell equations. We are interested in time-periodic, spatially localized solutions (breathers). For several scenarios (described by the assumptions on the coefficients g, h) we prove existence of breathers based on variational methods or bifurcation theory.

This is joint work with G. Brüll (Lund), P. Idzik (KIT), S. Kohler (KIT) and S. Ohrem (KIT).

Boundary value problems with rough boundary data

Jörg Seiler

Università degli Studi di Torino
joerg.seiler@unito.it

Abstract

We consider linear boundary value problems for higher-order parameter-elliptic equations, where the boundary data do not belong to the classical trace spaces. We employ a class of Sobolev spaces of mixed smoothness that admits a generalized boundary trace with values in Besov spaces of negative order. We prove unique solvability for rough

boundary data in the half-space and in sufficiently smooth domains. As an application, we show that the operator related to the linearized Cahn–Hilliard equation with dynamic boundary conditions generates a holomorphic semigroup in $L^p(\mathbb{R}^n_+) \times L^p(\mathbb{R}^{n-1})$. This is joint work with R. Denk, D. Ploß and S. Rau (all University of Konstanz).

Fluid flow on surfaces

Gieri Simonett

Vanderbilt University Nashville
gieri.simonett@vanderbilt.edu

Abstract

Of concern is the motion of an incompressible viscous fluid on compact surfaces without boundary. Local in time well-posedness is established in the framework of L_p-maximal regularity for initial values in critical spaces. It will be shown that the set of equilibria consists exactly of the Killing vector fields. Each equilibrium is stable and any solution starting close to an equilibrium converges at an exponential rate to a (possibly different) equilibrium. In case the surface is two-dimensional, it will be shown that any solution with divergence free initial value in L_2 exists globally and converges to an equilibrium. (Joint work with Mathias Wilke).

Large amplitude steady water waves with vorticity

Erik Wahlén

Lund University
erik.wahlen@math.lu.se

Abstract

I will talk about recent work on two-dimensional steady periodic gravity water waves with vorticity. In particular, I will present a recent formulation for general vorticity functions which allows for overhanging waves with interior stagnation points and is convenient for global bifurcation theory. In the special case of constant vorticity, more detailed information about the qualitative features of overhanging waves will be presented. This is based on joint work with J. Weber, E. Lokharu and F. Carvalho.

Some Remarks on the Viscosity Limit Problem

Emil Wiedemann

Universität Erlangen
emil.wiedemann@fau.de

Abstract

Historically, the equations of ideal fluid dynamics written down by Euler in 1757 were among the first partial differential equations ever to be formulated. It took decades to develop a refinement of Euler's model – the Navier-Stokes system – where friction effects within the fluid are taken into consideration. Yet, close-to-ideal fluids remain of great interest, as they are likely to display effects of turbulence. It is therefore an important question whether, as viscosity tends to zero, the solutions of the corresponding Navier-Stokes equations converge to the, or a, solution of the Euler system. We will present recent insights into this question in various settings, including shear flows, bounded domains, and peculiarities of the two-dimensional theory.

The role of cross-degeneracies in reaction-diffusion driven structure evolution

Michael Winkler

Universität Paderborn
michael.winkler@math.uni-paderborn.de

Abstract

Simple models for nutrient-oriented bacterial migration are compared. In particular, the potential of certain cross-degenerate diffusion mechanisms to adequately describe experimentally observed phenomena related to emergence and stabilization of structures are discussed. Resulting mathematical challenges are described and possible approaches outlined, both at levels of basic existence theories and at the stage of qualitative analysis.